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Network Analysis with the Help of Graph Theory

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ABSTRACT: Graph theory, a branch of Mathematics plays a vital rule in studying interdisciplinary subjects such as physics , Chemistry, Engineering etc. Study of the properties of electrical circuits with the help of graph theory is a growing trend in mathematical and electrical fields. Electrical circuits consists of nodes and branches which obeys Kirchhoff's current laws, Kirchhoff's voltage laws etc. There are various well known theorems such as Norton's theorem, Thevenin's theorem, Superposition theorem ,Millman Theorem etc for network analysis. In this paper, we try to analyze Millman's theorem with the help of Graph theorem.
Keywords: Graph theory , Electrical circuits, Branch current , Loop current, Adjacency matrix, Impedance matrix,.2000 A.M.S. subject classification: . 05C50, 11F27.

I. INTRODUCTION :

From every electrical circuit can find a graph . Therefore we can analyse electrical networks by using graph theory instead of existing well known network theorems such as Thevenin's theorem , Norton's theorem, Millman's theorem etc .

1.1 Graph: A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$. The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. So in a graph a vertex set is a set of points $\{x_1, x_2, x_3 \dots . x_n\}$and edge is a line which connects two points x_i and x_j .Such graphs are called undirected graphs. A directed graph is similar to an undirected graph except the edge set$E \in V \times V$.

1.2 ELECTRICAL CIRCUITS : An electrical circuit consists of internally connected elements viz resistors , capacitors, inductors, diodes, transistors etc . The behaviour of an electrical circuits generally depends upon two factors.

a) The characteristic of each of internally connected elements

b) The rule by which they are connected together.

The second factor gives a relation between electrical circuits with graph theory. A two terminal electrical element can be represented by an edge e_k . Associate with each edge there are two variables $V_k(t)$ and $i_k(t)$. The variable $V_k(t)$ is called the edge voltage and may be regarded as cross variable because it exist across the two end points. The other variable is called edge current and may be regarded as through variable because it flows through the edge. These variables must also obey the two laws of Kirchhoff's.

1.3 Kirchhoff's Current Law (KCL): For any lumped electrical network, at any time the net sum (taking into account the orientations) of all the currents leaving any node or vertex is zero. That is at r^{th} vertex of the corresponding digraph , we must have

$$\sum_{k=1}^{e} a_{rk} i_k(t) = 0$$

Where a_{rk} is the rk th entry of the incidence matrix A of G and $i_k(t)$ is the amount of current flowing through the k^{th} edge of G.

1.4 Kirchhoff's Voltage Law (KVL) : For any lumped electrical network, at any time the net sum (taking into account the orientations) of the voltages around a loop (i.e. circuit) is zero. In terms of the corresponding digraph , for the r^{th} circuit we must have

$$\sum_{k=1}^{e} b_{rk} v_k(t) = 0$$

Where b_{rk} is the rk th entry of the circuit matrix B of G and $V_k(t)$ is the amount of voltage across the k^{th} edge.

1.5 From Circuit to Graph :

A graph can be obtained from a circuit. We identify the graph G= (V, E) where V is the set of vertices and E is the set of edges. The edge between i^{th} and j^{th} vertices can be denoted by {i,j} ignoring the direction. Similarly the notation (i,j)

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can be used for oriented edges, where i is the start vertex and j is the end vertex. For example consider the circuit and its graph in the figure. There are five vertices and seven edges in the graph obtained from the given circuit. An edge is sometimes called branch and a vertex is called node in case of electrical circuits.

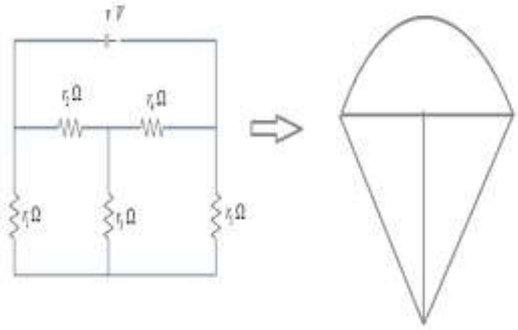


Fig: A circuit and its corresponding graph

1.6 MATRICES ASSOCIATED TO A GRAPH :

1.6.1. **Fundamental Tie set matrix (Fundamental loop matrix)** :

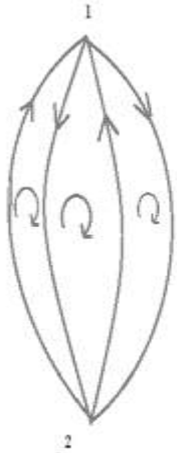
This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branch of trees). Here we assume that the direction of loop currents and direction of the link is same.

So in the matrix

$b_{ij} = 1$, if the branch b_j in the fundamental loop i ane their reference direction oriented same

$b_{ij} = -1$ if the branch b_j in the fundamental loop i ane their reference direction oriented opposite

$b_{ij} = 0$ if the branch b_j is not in the fundamental loop i



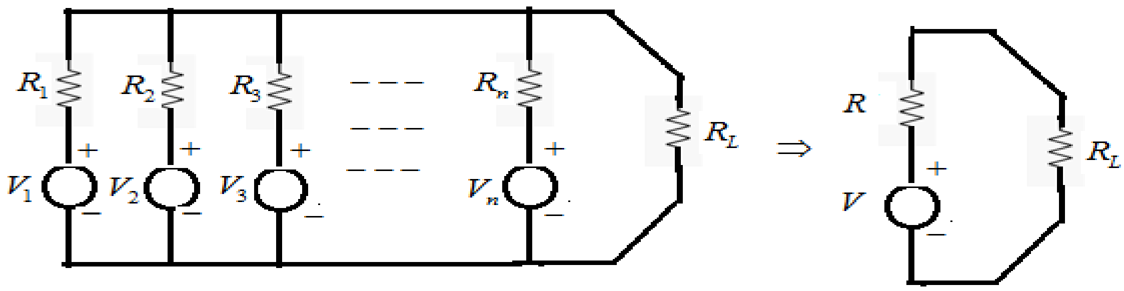
$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

1.6.2. **Branch Impedance matrix [Z_b]**:

It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entries are zero.

II. MILLMAN'S THEOREM

With the help of this theorem any numbers of parallel voltage sources can be reduce to one equivalent source .Let us consider a number of parallel voltage sources $V_1, V_2, V_3, \ldots, V_n$ having internal resistances $R_1, R_2, R_3, \ldots, R_n$ respectively .The arrangement can be replaced by a single equivalent voltage source V in series with an equivalent series resistance R as shown in the figure .



By Millman's theorem the equivalent voltage and resistance of the reduced circuit is given by the relations

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm V_3 G_3 \pm \ldots \pm V_n G_n}{G_1 + G_2 + G_3 + \ldots + G_n} \quad \text{where} \quad R = \frac{1}{G} = \frac{1}{G_1 + G_2 + G_3 + \ldots + G_n} .$$

Let us convert the voltage sources into current sources as follows

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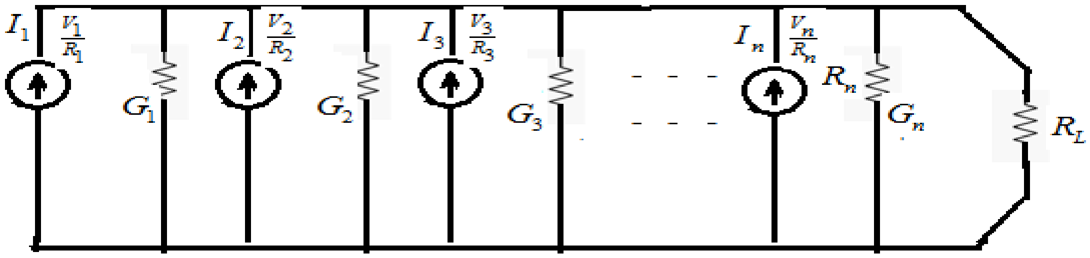
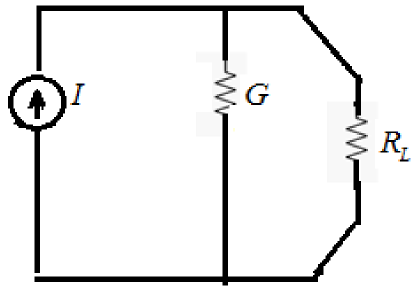
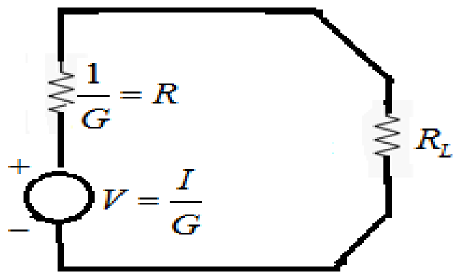


If I be the resultant current of the parallel current sources where G is the equivalent conductance .
$I = I_1 + I_2 + I_3 + ... + I_n$ and $G = G_1 + G_2 + G_3 + ... + G_n$.
Then the equivalent circuit will be



Now converting the current sources to equivalent voltage sources as



Thus we have $V = \dfrac{I}{G} = \dfrac{\pm I_1 \pm I_2 \pm I_3 \pm ..... \pm I_n}{G_1 + G_2 + G_3 + ... + G_n}$

Also $R = \dfrac{1}{G} = \dfrac{1}{G_1 + G_2 + G_3 + ... + G_n}$

and $V = \dfrac{\pm \frac{V_1}{R_1} \pm \frac{V_2}{R_2} \pm \frac{V_3}{R_3} \pm ..... \pm \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}}$

Where R is the equivalent resistance connected with the equivalent voltage source in series .

III. OUR PROBLEM

With the help of Millman's theorem find the current through $4r\ \Omega$ of the following circuit .

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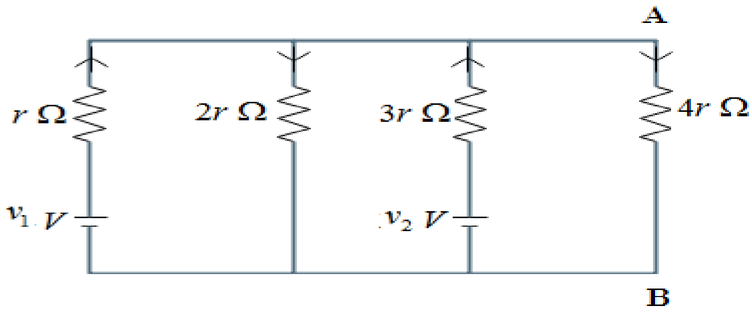
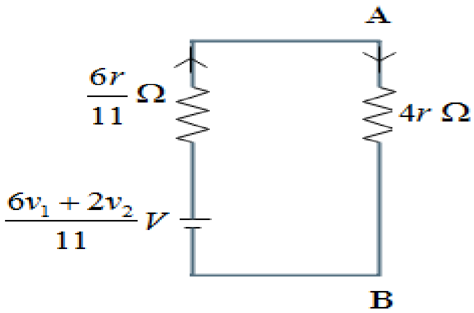


Voltage across the terminal $A-B=V_{AB}=\dfrac{\dfrac{E_1}{R_1}+\dfrac{E_2}{R_2}+\dfrac{E_3}{R_3}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}=\dfrac{\dfrac{v_1}{r}+\dfrac{0}{2r}+\dfrac{v_2}{3r}}{\dfrac{1}{r}+\dfrac{1}{2r}+\dfrac{1}{3r}}=\dfrac{6v_1+2v_2}{11}$

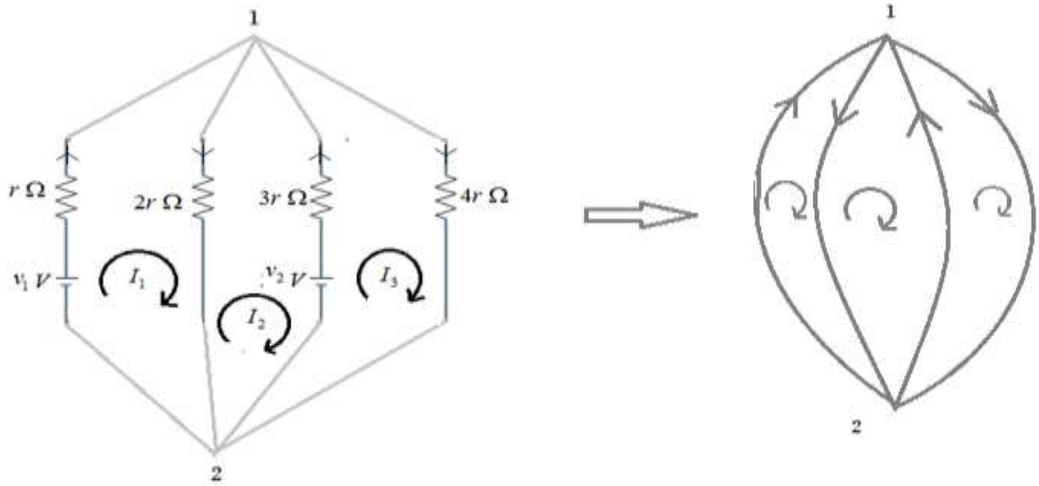
And equivalent resistance $\mathrm{R}_{eq}=\dfrac{1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}}=\dfrac{1}{\dfrac{1}{r}+\dfrac{1}{2r}+\dfrac{1}{3r}}=\dfrac{6r}{11}ohms$



Therefore by Millman's theorem the current through $4r\ \Omega$ is obtained as

$$I_{4r\,\Omega}=\frac{V_{Th}}{\mathrm{R}_{eq}+4r}=\frac{\dfrac{6v_1+2v_2}{11}}{\dfrac{6r}{11}+4r}=\frac{3v_1+v_2}{25r}A$$

Now let us study this circuit with the help of graph theory.
The graph of the given circuit is as given below .This graph has two vertices 1 and 2 and four edges .



$$[B]=\begin{bmatrix}1&1&0&0\\0&-1&-1&0\\0&0&1&1\end{bmatrix}\qquad [V_s]=\begin{bmatrix}v_1\\0\\v_2\\0\end{bmatrix}\qquad [Z_b]=\begin{bmatrix}r&0&0&0\\0&2r&0&0\\0&0&3r&0\\0&0&0&4r\end{bmatrix}$$

$$[B][V_s]=\begin{bmatrix}1&1&0&0\\0&-1&-1&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}v_1\\0\\v_2\\0\end{bmatrix}=\begin{bmatrix}v_1\\-v_2\\v_2\end{bmatrix}$$

$$[B][Z_b]=\begin{bmatrix}1&1&0&0\\0&-1&-1&0\\0&0&1&1\end{bmatrix}\begin{bmatrix}r&0&0&0\\0&2r&0&0\\0&0&3r&0\\0&0&0&4r\end{bmatrix}=\begin{bmatrix}r&2r&0&0\\0&-2r&-3r&0\\0&0&3r&4r\end{bmatrix}$$

$$[B][Z_b][B']=\begin{bmatrix}r&2r&0&0\\0&-2r&-3r&0\\0&0&3r&4r\end{bmatrix}\begin{bmatrix}1&0&0\\1&-1&0\\0&-1&1\\0&0&1\end{bmatrix}=\begin{bmatrix}3r&-2r&0\\-2r&5r&-3r\\0&-3r&7r\end{bmatrix}$$

By Kirchhoff ‘s Voltage Law we have

$$[B][Z_b][B'][I]=-[B][V_s]$$

$$\Rightarrow\begin{bmatrix}3r&-2r&0\\-2r&5r&-3r\\0&-3r&7r\end{bmatrix}\begin{bmatrix}I_1\\I_2\\I_3\end{bmatrix}=-\begin{bmatrix}v_1\\-v_2\\v_2\end{bmatrix}$$

$$\therefore 3rI_1-2rI_2=-v_1$$

$$-2rI_1+5rI_2-3rI_3=-v_2$$

$$-3rI_2+7rI_3=-v_2$$

$$\therefore \quad I_1 = \frac{-13v_1 + 4v_2}{25r} A \quad , \quad I_2 = \frac{-7v_1 + 6v_2}{25r} A \quad , \quad I_3 = \frac{-3v_1 - v_2}{25r} A$$

Now we have to find the branch current of the circuit

$$[i_b] = [B'][I_L]$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-13v_1 + 4v_2}{25r} \\ \frac{-7v_1 + 6v_2}{25r} \\ \frac{-3v_1 - v_2}{25r} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \frac{-13v_1 + 4v_2}{25r} \\ -\frac{6v_1 + 2v_2}{25r} \\ \frac{4v_1 - 7v_2}{25r} \\ -\frac{3v_1 + v_2}{25r} \end{bmatrix}$$

$$\therefore i_1 = \frac{-13v_1 + 4v_2}{25r}, i_2 = -\frac{6v_1 + 2v_2}{25r}, i_3 = \frac{4v_1 - 7v_2}{25r}, i_4 = -\frac{3v_1 + v_2}{25r}$$

Therefore current across the resistance $4r\,\Omega$ is $i_4 = -\frac{3v_1 + v_2}{25r}$ which is same as the current obtain by Millman's theorem. (Negative sign shows that orientation of the current is in opposite direction)

IV. CONCLUSION:

In view of above from our study it is clear that graph theoretic approach is the best effective method for network analysis or alternatively we can conclude that Millman's theorems can be replaced by graph theoretic model

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